

STUDY OF REACTIVE POWER COMPENSATION USING STATCOM

A PROJECT SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENT FOR THE DEGREE OF

Bachelor of Technology

in

Electrical Engineering

By

Abhijeet Barua (107EE015)

Pradeep Kumar (107EE050)



Department of Electrical Engineering

National Institute of Technology

Rourkela-769008, Orissa

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CERTIFICATE

This is to certify that the Project entitled “**STUDY OF REACTIVE POWER COMPENSATION USING STATCOM**” submitted by Abhijeet Barua and Pradeep Kumar in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Electrical Engineering at National Institute of Technology, Rourkela (Deemed University), is an authentic work carried out by them under my supervision and guidance.

Date:

(Prof. P. C. Panda)

Place: Rourkela

Department of Electrical Engineering

NIT, Rourkela

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ABSTRACT

The study of shunt connected FACTS devices is a connected field with the problem of reactive power compensation and better mitigation of transmission related problems in today's world. In this paper we study the shunt operation of FACTS controller, the STATCOM, and how it helps in the better utilization of a network operating under normal conditions. First we carry out a literature review of many papers related to FACTS and STATCOM, along with reactive power control. Then we look at the various devices being used for both series and shunt compensation. The study of STATCOM and its principles of operation and control, including phase angle control and PWM techniques, are carried out. We also delve into the load flow equations which are necessary for any power system solution and carry out a comprehensive study of the Newton Raphson method of load flow. Apart from this, we also carry out a study of the transient stability of power systems, and how it is useful in determining the behavior of the system under a fault. As an example, a six bus system is studied using the load flow equations and solving them. First this is done without the STATCOM and then the STATCOM is implemented and the characteristics of the rotor angle graph along with faults at various buses are seen. In this thesis, it is tried to show the application of STATCOM to a bus system and its effect on the voltage and angle of the buses. Next the graphs depicting the implemented STATCOM bus are analyzed and it is shown that the plots of the rotor angles show a changed characteristic under the influence of the STATCOM.

INTRODUCTION

Power Generation and Transmission is a complex process, requiring the working of many components of the power system in tandem to maximize the output. One of the main components to form a major part is the reactive power in the system. It is required to maintain the voltage to deliver the active power through the lines. Loads like motor loads and other loads require reactive power for their operation. To improve the performance of ac power systems, we need to manage this reactive power in an efficient way and this is known as reactive power compensation. There are two aspects to the problem of reactive power compensation: load compensation and voltage support. Load compensation consists of improvement in power factor, balancing of real power drawn from the supply, better voltage regulation, etc. of large fluctuating loads. Voltage support consists of reduction of voltage fluctuation at a given terminal of the transmission line. Two types of compensation can be used: series and shunt compensation. These modify the parameters of the system to give enhanced VAR compensation. In recent years, static VAR compensators like the STATCOM have been developed. These quite satisfactorily do the job of absorbing or generating reactive power with a faster time response and come under Flexible AC Transmission Systems (FACTS). This allows an increase in transfer of apparent power through a transmission line, and much better stability by the adjustment of parameters that govern the power system i.e. current, voltage, phase angle, frequency and impedance.

1.1 Reactive Power

Reactive power is the power that supplies the stored energy in reactive elements. Power, as we know, consists of two components, active and reactive power. The total sum of active and reactive power is called as apparent power.

In AC circuits, energy is stored temporarily in inductive and capacitive elements, which results in the periodic reversal of the direction of flow of energy between the source and the load. The average power after the completion of one whole cycle of the AC waveform is the real power, and this is the usable energy of the system and is used to do work, whereas the portion of power flow which is temporarily stored in the form of magnetic or electric fields and flows back and forth in the transmission line due to inductive and capacitive network elements is known as reactive power. This is the unused power which the system has to incur in order to transmit power.

Inductors (reactors) are said to store or absorb reactive power, because they store energy in the form of a magnetic field. Therefore, when a voltage is initially applied across a coil, a magnetic field builds up, and the current reaches the full value after a certain period of time. This in turn causes the current to lag the voltage in phase.

Capacitors are said to generate reactive power, because they store energy in the form of an electric field. Therefore when current passes through the capacitor, a charge is built up to produce the full voltage difference over a certain period of time. Thus in an AC network the voltage across the capacitor is always changing. Since, the capacitor tends to oppose this change; it causes the voltage to lag behind current in phase.

In an inductive circuit, we know the instantaneous power to be:

$$p = V_{\max} I_{\max} \cos \omega t \cos(\omega t - \theta)$$

$$p = \frac{V_{\max} I_{\max}}{2} \cos \theta (1 + \cos 2\omega t) + \frac{V_{\max} I_{\max}}{2} \sin \theta \sin 2\omega t$$

The instantaneous reactive power is given by:

$$\frac{V_{\max} I_{\max}}{2} \sin \theta \sin 2\omega t$$

Where:

p = instantaneous power

V_{\max} = Peak value of the voltage waveform

I_{\max} = Peak value of the current waveform

ω = Angular frequency

= $2\pi f$ where f is the frequency of the waveform.

t = Time period

θ = Angle by which the current lags the voltage in phase

From here, we can conclude that the instantaneous reactive power pulsates at twice the system frequency and its average value is zero and the maximum instantaneous reactive power is given by:

$$Q = |V| |I| \sin \theta$$

The zero average does not necessarily mean that no energy is flowing, but the actual amount that is flowing for half a cycle in one direction, is coming back in the next half cycle.

1.2 Compensation Techniques

The principles of both shunt and series reactive power compensation techniques are described below:

1.2.1 Shunt compensation

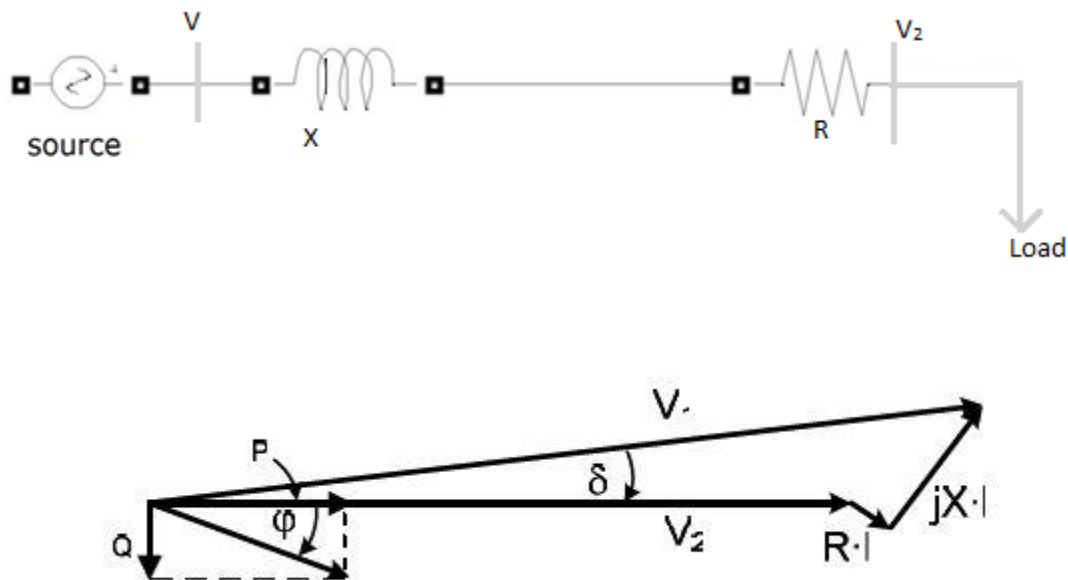


Fig 1.1

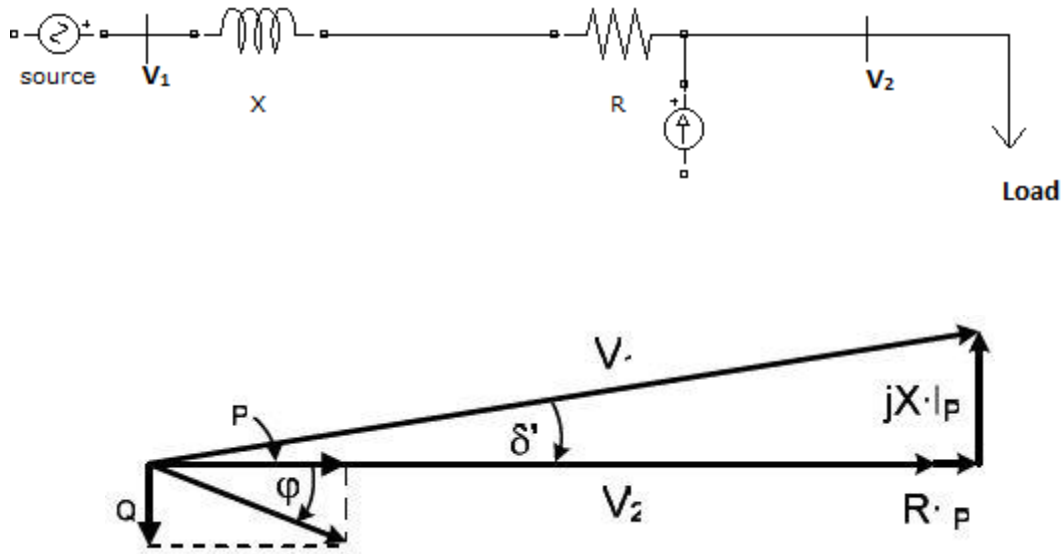


Fig 1.2

The figure 1.1 comprises of a source V_1 , a power line and an inductive load. The figure 1.1 shows the system without any type of compensation. The phasor diagram of these is also shown above. The active current I_p is in phase with the load voltage V_2 . Here, the load is inductive and hence it requires reactive power for its proper operation and this has to be supplied by the source, thus increasing the current from the generator and through the power lines. Instead of the lines carrying this, if the reactive power can be supplied near the load, the line current can be minimized, reducing the power losses and improving the voltage regulation at the load terminals. This can be done in three ways: 1) A voltage source. 2) A current source. 3) A capacitor.

In this case, a current source device is used to compensate I_q , which is the reactive component of the load current. In turn the voltage regulation of the system is improved and the reactive current component from the source is reduced or almost eliminated. This is in case of lagging compensation. For leading compensation, we require an inductor.

Therefore we can see that, a current source or a voltage source can be used for both leading and lagging shunt compensation, the main advantages being the reactive power generated is independent of the voltage at the point of connection.

1.2.2 Series compensation

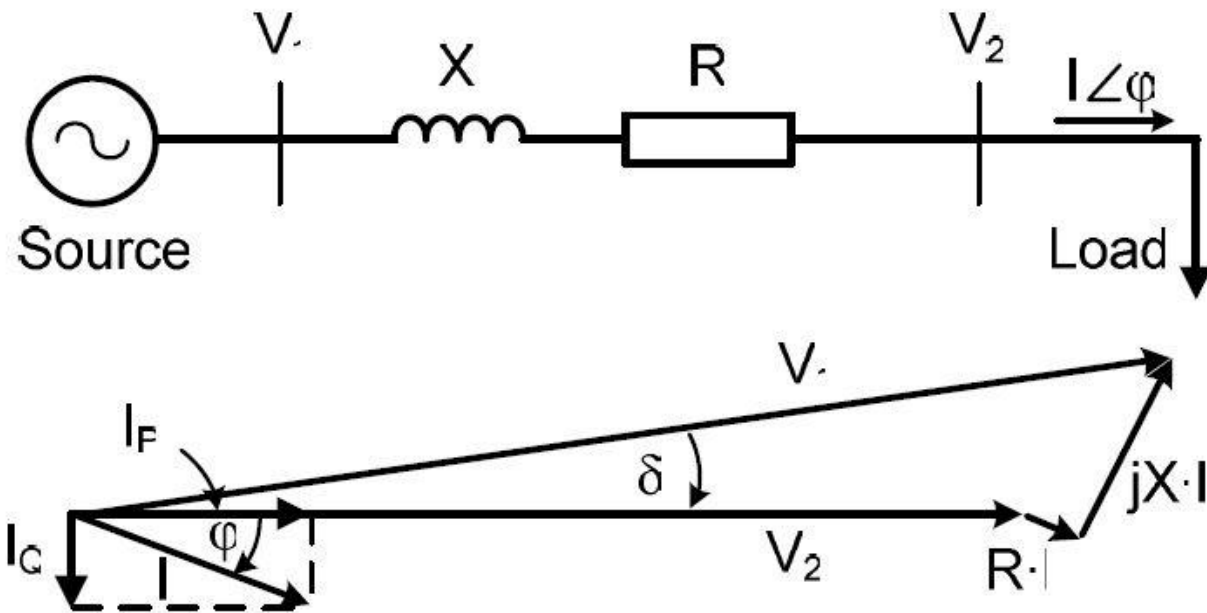


Fig 1.3

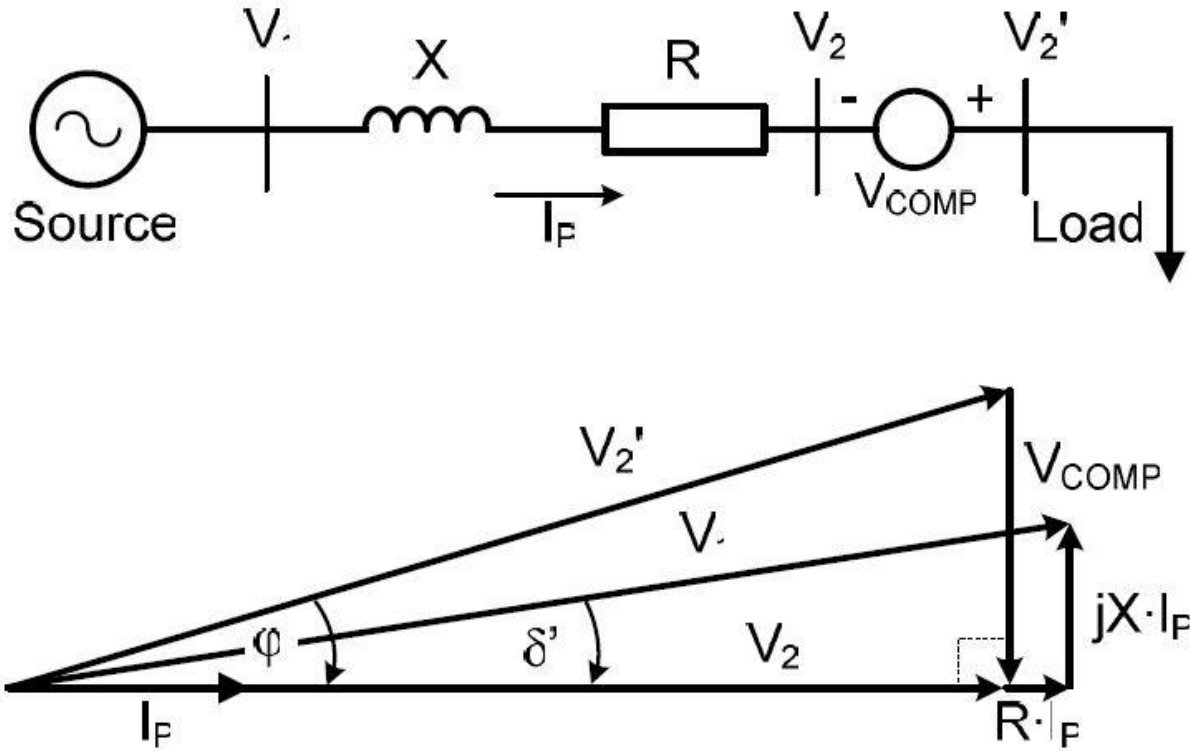


Fig 1.4

Series compensation can be implemented like shunt compensation, i.e. with a current or a voltage source as shown in figure 1.4. We can see the results which are obtained by series compensation through a voltage source and it is adjusted to have unity power factor at V_2 . However series compensation techniques are different from shunt compensation techniques, as capacitors are used mostly for series compensation techniques. In this case, the voltage V_{comp} has been added between the line and the load to change the angle V_2' . Now, this is the voltage at the load side. With proper adjustment of the magnitude of V_{comp} , unity power factor can be reached at V_2 .

1.3 FACTS devices used

Flexible AC transmission system or FACTS devices used are:

1) VAR generators.

- a) Fixed or mechanically switched capacitors.
- b) Synchronous condensers.
- c) Thyristorized VAR compensators.
 - (i) Thyristors switched capacitors (TSCs).
 - (ii) Thyristor controlled reactor (TCRs).
 - (iii) Combined TSC and TCR.
 - (iv) Thyristor controlled series capacitor (TCSC).

2) Self Commutated VAR compensators.

- a) Static synchronous compensators (STATCOMs).
- b) Static synchronous series compensators (SSSCs).
- c) Unified power flow controllers (UPFCs).
- d) Dynamic voltage restorers (DVRs).

1.4 Need for Reactive power compensation.

The main reason for reactive power compensation in a system is: 1) the voltage regulation; 2) increased system stability; 3) better utilization of machines connected to the system; 4) reducing losses associated with the system; and 5) to prevent voltage collapse as well as voltage sag. The impedance of transmission lines and the need for lagging VAR by most

machines in a generating system results in the consumption of reactive power, thus affecting the stability limits of the system as well as transmission lines. Unnecessary voltage drops lead to increased losses which needs to be supplied by the source and in turn leading to outages in the line due to increased stress on the system to carry this imaginary power. Thus we can infer that the compensation of reactive power not only mitigates all these effects but also helps in better transient response to faults and disturbances. In recent times there has been an increased focus on the techniques used for the compensation and with better devices included in the technology, the compensation is made more effective. It is very much required that the lines be relieved of the obligation to carry the reactive power, which is better provided near the generators or the loads. Shunt compensation can be installed near the load, in a distribution substation or transmission substation.

2.1 Static Shunt Compensator: STATCOM

One of the many devices under the FACTS family, a STATCOM is a regulating device which can be used to regulate the flow of reactive power in the system independent of other system parameters. STATCOM has no long term energy support on the dc side and it cannot exchange real power with the ac system. In the transmission systems, STATCOMs primarily handle only fundamental reactive power exchange and provide voltage support to buses by modulating bus voltages during dynamic disturbances in order to provide better transient characteristics, improve the transient stability margins and to damp out the system oscillations due to these disturbances.

A STATCOM consists of a three phase inverter (generally a PWM inverter) using SCRs, MOSFETs or IGBTs, a D.C capacitor which provides the D.C voltage for the inverter, a link reactor which links the inverter output to the a.c supply side, filter components to filter out the high frequency components due to the PWM inverter. From the d.c. side capacitor, a three phase voltage is generated by the inverter. This is synchronized with the a.c supply. The link inductor links this voltage to the a.c supply side. This is the basic principle of operation of STATCOM.

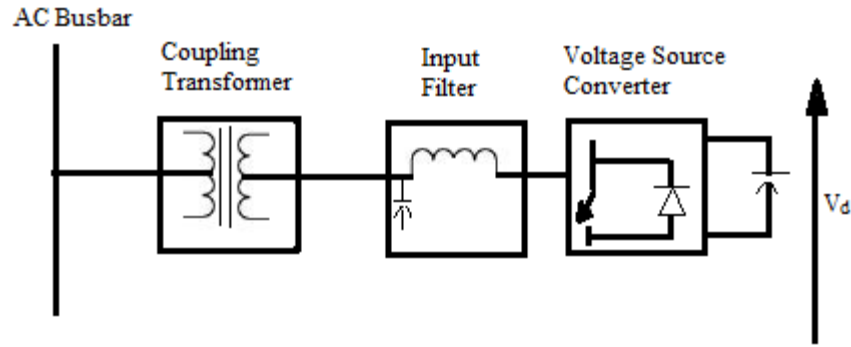


Fig 2.1

For two AC sources which have the same frequency and are connected through a series inductance, the active power flows from the leading source to the lagging source and the reactive power flows from the higher voltage magnitude source to the lower voltage magnitude source. The phase angle difference between the sources determines the active power flow and the voltage magnitude difference between the sources determines the reactive power flow. Thus, a STATCOM can be used to regulate the reactive power flow by changing the magnitude of the VSC voltage with respect to source bus voltage.

2.2 Phase angle control

In this case the quantity controlled is the phase angle δ . The modulation index “m” is kept constant and the fundamental voltage component of the STATCOM is controlled by changing the DC link voltage. By further charging of the DC link capacitor, the DC voltage will be increased, which in turn increases the reactive power delivered or the reactive power absorbed by the STATCOM. On the other hand, by discharging the DC link capacitor, the reactive power delivered is decreased in capacitive operation mode or the reactive power absorbed by the STATCOM in an inductive power mode increases.

For both capacitive and inductive operations in steady-state, the STATCOM voltage lags behind AC line voltage ($\delta > 0$).

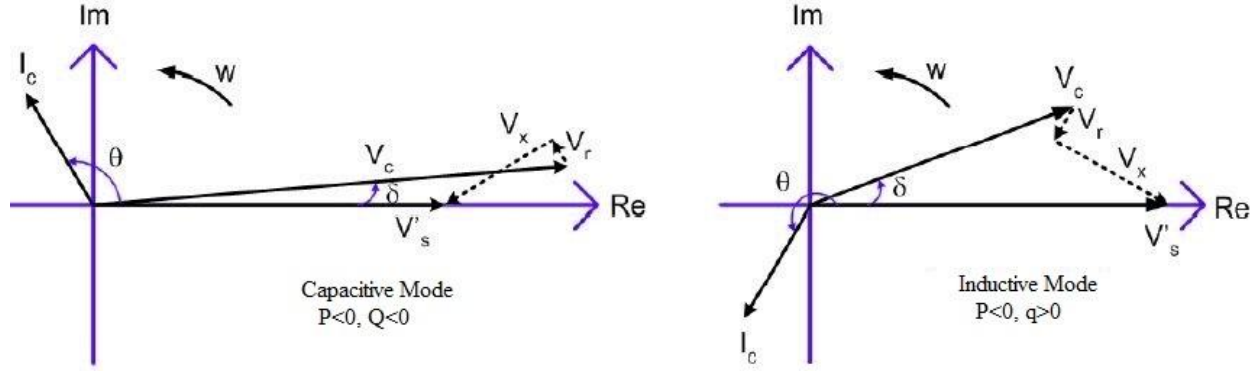


Fig 2.2

By making phase angle δ negative, power can be extracted from DC link. If the STATCOM becomes lesser than the extracted power, P_c in becomes negative and STATCOM starts to deliver active power to the source. During this transient state operation, V_d gradually decreases.

The phasor diagrams which illustrating power flow between the DC link in transient state and the ac supply is shown in above Fig.

For a phase angle control system, the open loop response time is determined by the DC link capacitor and the input filter inductance. The inductance is applied to filter out converter harmonics and by using higher values of inductance; the STATCOM current harmonics is minimized.

The reference reactive power (Q_{ref}) is compared with the measured reactive power (Q). The reactive power error is sent as the input to the PI controller and the output of the PI controller determines the phase angle of the STATCOM fundamental voltage with respect to the source voltage.

2.3 PWM Techniques used in STATCOM

Sinusoidal PWM technique

We use sinusoidal PWM technique to control the fundamental line to-line converter voltage. By comparing the three sinusoidal voltage waveforms with the triangular voltage waveform, the three phase converter voltages can be obtained.

The fundamental frequency of the converter voltage i.e. f_1 , modulation frequency, is determined by the frequency of the control voltages, whereas the converter switching frequency is determined by the frequency of the triangular voltage i.e. f_s , carrier frequency. Thus, the modulating frequency f_1 is equal to the supply frequency in STATCOM.

The Amplitude modulation ratio, m_a is defined as:

$$m_a = \frac{V_{control}}{V_{tri}}$$

Where $V_{control}$ is the peak amplitude of the control voltage waveform and V_{tri} is the peak amplitude of the triangular voltage waveform. The magnitude of triangular voltage is maintained constant and the $V_{control}$ is allowed to vary.

The range of SPWM is defined for $0 \leq m_a \leq 1$ and over modulation is defined for $m_a > 1$.

The frequency modulation ratio m_f is defined as:

$$m_f = \frac{f_s}{f_i}$$

The frequency modulation ratio, m_f , should have odd integer values for the formation of odd and half wave symmetric converter line-to-neutral voltage(V_{A0}). Thus, even harmonics are eliminated from the V_{A0} waveform. Also, to eliminate the harmonics we choose odd multiples of 3 for m_f .

The converter output harmonic frequencies can be given as:

$$f_h = (jm_f \pm k)f_1$$

The relation between the fundamental component of the line-to-line voltage (V_{A0}) and the amplitude modulation ratio m_a can be gives as:

$$V_{A0} = m_a \frac{V_d}{2}, m_a \leq 1$$

From which, we can see that V_{A0} varies linearly with respect to m_a , irrespective of m_f .

The fundamental component converter line-to-line voltage can be expressed as:

$$V_{LL1} = \frac{\sqrt{3}}{2\sqrt{2}} m_a V_d; m_a \leq 1$$

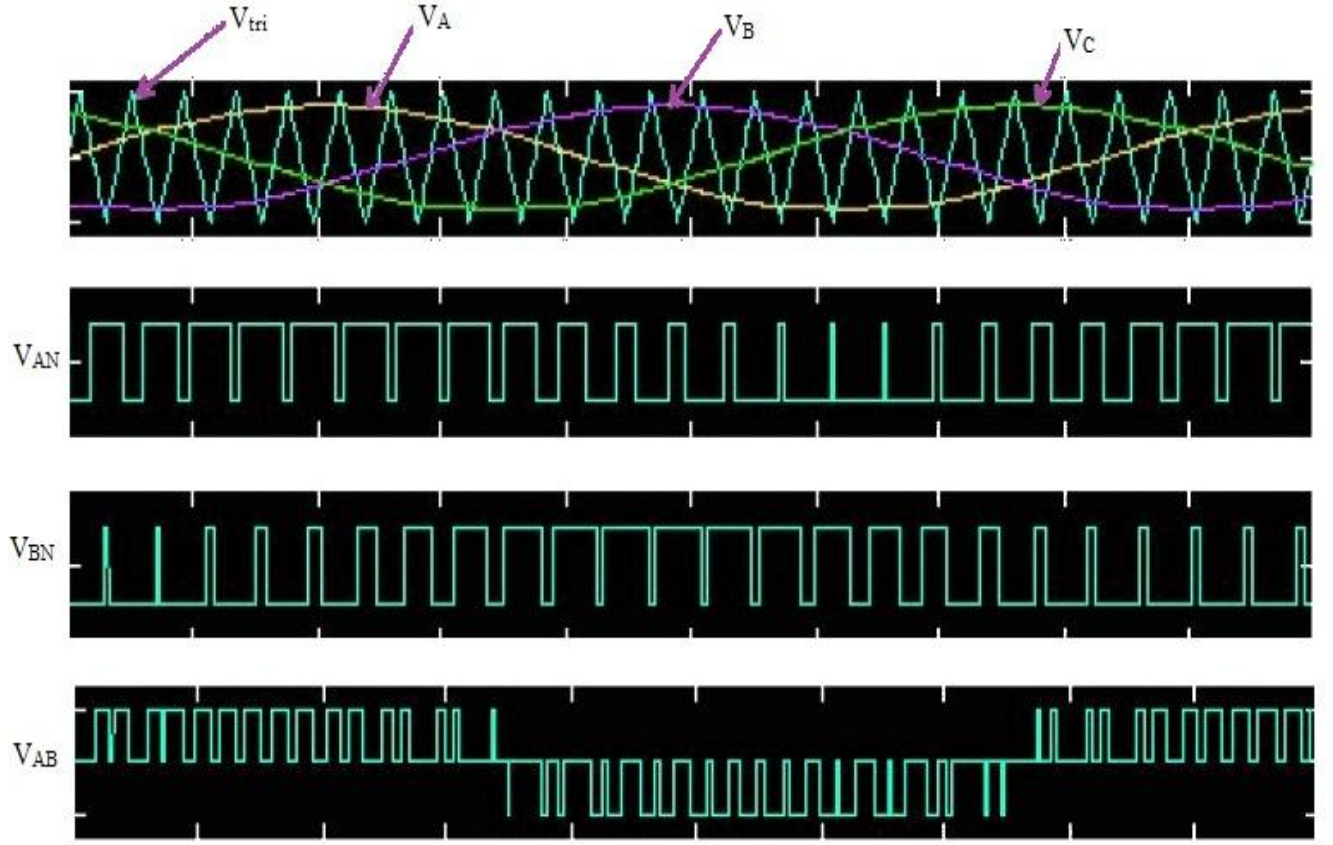


Fig 2.3

In this type of PWM technique, we observe switching harmonics in the high frequency range around the switching frequency and its multiples in the linear range. From above equation, we can see that the amplitude of the fundamental component of the converter line-to-line voltage is $0.612m_a V_d$. But for square wave operation, we know the amplitude to be $0.78V_d$. Thus, in the linear range the maximum amplitude of fundamental frequency component is reduced. This can be solved by over modulation of the converter voltage waveform, which can increase the harmonics in the sidebands of the converter voltage waveform. Also, the amplitude of V_{LL1} varies nonlinearly with m_a and also varies with m_f in over modulation as given

In a Constant DC Link Voltage Scheme the STATCOM regulates the DC link voltage value to a fixed one in all modes of operation. This fixed value is determined by the peak STATCOM fundamental voltage from the full inductive mode of operation to full capacitive mode at minimum and maximum voltage supply. Therefore, for $0 \leq m_a \leq 1$;
The fundamental voltage is varied by varying m_a in the linear range.

3.1 Study of Load Flow Analysis

Load-flow studies are very common in power system analysis. Load flow allows us to know the present state of a system, given previous known parameters and values. The power that is flowing through the transmission line, the power that is being generated by the generators, the power that is being consumed by the loads, the losses occurring during the transfer of power from source to load, and so on, are iteratively decided by the load flow solution, or also known as power flow solution. In any system, the most important quantity which is known or which is to be determined is the voltage at different points throughout the system. Knowing these, we can easily find out the currents flowing through each point or branch. This in turn gives us the quantities through which we can find out the power that is being handled at all these points.

In earlier days, small working models were used to find out the power flow solution for any network. Because computing these quantities was a hard task, the working models were not very useful in simulating the actual one. It's difficult to analyze a system where we need to find out the quantities at a point very far away from the point at which these quantities are known. Thus we need to make use of iterative mathematical solutions to do this task, due to the fact that there are no finite solutions to load flow. The values more often converge to a particular value, yet do not have a definite one. Mathematical algorithms are used to compute the unknown quantities from the known ones through a process of successive trial and error methods and consequently produce a result. The initial values of the system are assumed and with this as

input, the program computes the successive quantities. Thus, we study the load flow to determine the overloading of particular elements in the system. It is also used to make sure that the generators run at the ideal operating point, which ensures that the demand will be met without overloading the facilities and maintain them without compromising the security of the system nor the demand.

The objective of any load-flow analysis is to produce the following information:

- Voltage magnitude and phase angle at each bus.
- Real and reactive power flowing in each element.
- Reactive power loading on each generator.

3.2 Types of Buses

Generally in an a.c. system, we have the variables like voltage, current, power and impedance. Whereas in a dc system, we have just the magnitude component of all these variables due to the static nature of the system, this is not the case in ac systems. AC systems bring one more component to the forefront, that of time. Thus any quantity in an AC system is described by two components: the magnitude component and the time component. For the magnitude we have the RMS value of the quantity, whereas for the time we take the phase angle component. Thus voltage will have a magnitude and a phase angle. Hence when we solve for the currents, we will get a magnitude and a phase angle. These two when combined, will give the power for the system, which will contain a real and a reactive term.

The actual variables that are given as inputs to the buses and the operating constraints that govern the working of each bus decide the types of buses. Thus we have the two main types of buses: the load bus and the generator bus. At the load bus, the variables that are already

specified are the real power (P) and the reactive power (Q) consumed by the load. The variables which are to be found out are voltage magnitude (V) and the phase angle of this voltage (δ). Hence load bus is also called PQ bus in power systems.

At the generator bus, the variables which are specified are the real power being generated (P) and the voltage at which this generation is taking place (V). The variables which are to be found out are the generator reactive power (Q) and the voltage phase angle (δ). This is done so for convenience as the power needs of the system need to be balanced as well as the operational control of the generator needs to be optimized.

Apart from these two we have the slack bus which is responsible for providing the losses in the whole system and the transmission lines and thus is specified by the variables voltage magnitude (V) and angle (δ).

If we are given any of the two inputs of the system, along with the fixed parameters like impedance of the transmission lines as well as that of the system, and system frequency, then using mathematical iterations we can easily find out the unknown variables. Thus the operating state of the system can be determined easily knowing the two variables. The variables to be specified and the variables to be computed are given below.

Type of bus	Specified quantities	Calculated quantities
Generator bus	Real power (P)	Reactive power (Q)
(PV Bus)	Voltage magnitude (V)	Voltage angle (δ)
Load bus	Real power (P)	Voltage magnitude (V)
(PQ Bus)	Reactive power (Q)	Voltage angle (δ)
Slack bus	Voltage magnitude (V)	Real power (P)
	Voltage angle (δ)	Reactive power (Q)

3.3 Load Flow Equations and their Solutions

3.3.1 Development of Load Flow Equations

The real and reactive power components for any bus p can be used as:

$$P_p - jQ_p = V_p^* I_p$$

$$\Rightarrow I_p = \frac{P_p - jQ_p}{V_p^*}$$

Now the nodal current equations for a n-bus system can be written as

$$I_p = \sum_{q=1}^n Y_{pq} V_q, \quad p = 1, 2, 3, \dots, n;$$

$$I_p = Y_{pp} V_p + \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q$$

$$\Rightarrow V_p = \frac{I_p}{Y_{pp}} - \frac{1}{Y_{pp}} \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q$$

Now,

$$V_p^* I_p = P_p - jQ_p$$

$$\Rightarrow I_p = \frac{P_p - jQ_p}{V_p^*}$$

Substituting for I_p in the above equation,

$$V_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{V_p^*} - \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q \right], \quad p = 1, 2, 3, \dots, n;$$

We substitute I_p by active and reactive power, because the quantities are usually specified in a power system.

3.3.2 Load Flow Equation Solution Methods

To start with by solving the load flow equations, we first assume values for the unknown variables in the bus system. For instance, let us suppose that the unknown variables are the magnitude of the voltages and their angles at every bus except the Slack bus, which makes them the load bus or the PQ bus. In this case, we assume the initial values of all voltage angles as zero and the magnitude as 1p.u. Meaning, we choose a flat voltage profile. We then put these assumed values in our power flow equations, knowing that these values don't represent the actual system, even though it should have been describing its state. So, now we iterate this process of putting in the values of voltage magnitudes and angles and replacing them with a better set. So, as the flat voltage profile keeps converging to the actual values of the magnitudes and angles, the mismatch between the P and Q will reduce. Depending on the number of iterations we use and our requirements we can end the process with values close to the actual value. This process is called as the iterative solution method.

The final equations derived in the previous section are the load flow equations where bus voltages are the variables. It can be seen that these equations are nonlinear and they can be solved using iterative methods like:

- 1) Gauss-Seidel method
- 2) Newton-Raphson method

3.3.2.1 Gauss-Seidel method

The Gauss-Seidel method is based on substituting nodal equations into each other. Its convergence is said to be Monotonic. The iteration process can be visualized for two equations:

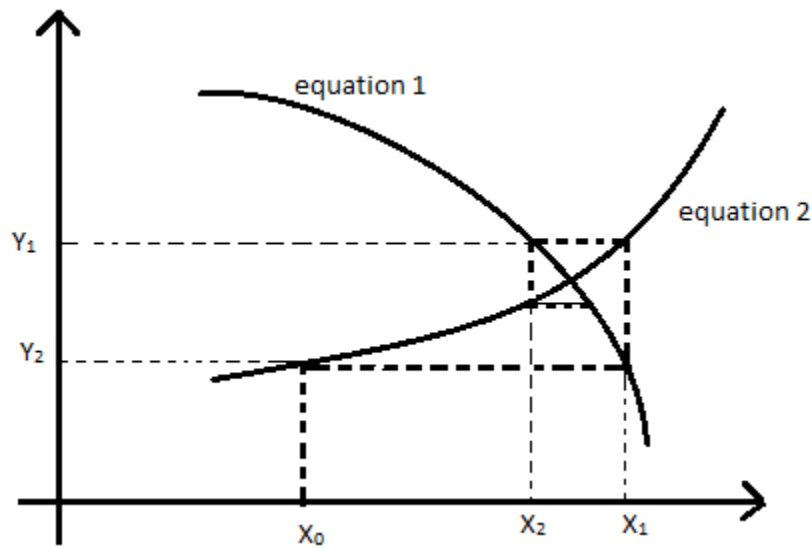


Fig 3.1

Although not the best load-flow method, Gauss-Seidel is the easiest to understand and was the most widely used technique until the early 1970s. Here, we use the Newton-Raphson method which is the most efficient load-flow algorithm.

3.3.2.2 Newton-Raphson (N-R) Method

Newton-Raphson algorithm is based on the formal application of a well-known algorithm for the solution of a set of simultaneous non-linear equations of the form:

$$[F(\mathbf{x})] = [0]$$

Where: $[F(\mathbf{x})]$ is a vector of functions: $f_1 \dots f_n$ in the variables $x_1 \dots x_n$.

The expression described above will not become equal zero until the N-R process has converged and the iterations been performed, assuming the initial set of values x_1, x_2, \dots, x_n . In the load-flow problem, where the x 's are voltage magnitude and phase angle at all load buses and voltage phase angles at all generator buses i.e., angles at all buses except slack and $|V|$ for all PQ buses.

The equations for load flow problem which can be solved by using N-R method can be derived as:

$$P_p - jQ_p = V_p^* I_p = V_p^* \sum_{q=1}^n Y_{pq} V_q$$

Let,

$$V_p = e_p + jf_p \quad \text{and} \quad Y_{pq} = G_{pq} - jB_{pq}$$

$$\begin{aligned} P_p - jQ_p &= (e_p + jf_p)^* \sum_{q=1}^n (G_{pq} - jB_{pq})(e_q + jf_q) \\ &= (e_p - jf_p) \sum_{q=1}^n (G_{pq} - jB_{pq})(e_q + jf_q) \end{aligned}$$

Separating the real and imaginary parts, we have:

$$P_p = \sum_{q=1}^n [e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq})]$$

And,

$$Q_p = \sum_{q=1}^n [f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})]$$

Also,

$$|V_p|^2 = e_p^2 + f_p^2$$

The three sets of equations above are the load flow equations for the N-R method and we can see that they are non-linear in terms of real and imaginary components of nodal voltages. The left hand quantities i.e. P_p, Q_p for a load bus and P_p and $|V_p|$ for a generator bus are specified and e_p and f_p are unknown quantities. For an n-bus system, the number of unknowns are (2n-1) because the voltage at the slack bus is known and is kept fixed both in magnitude and phase. Thus, if bus 1 is taken as the slack bus, the unknowns are $e_2, e_3, \dots, e_{n-1}, e_n$ and $f_2, f_3, \dots, f_{n-1}, f_n$.

Thus to solve all these variables, we need to solve all the 2(n-1) equations.

The Newton-Raphson method helps us to replace a set of nonlinear power-flow equations with a linear set, using Taylor's series expansion. The mathematical background for this method is as follows:

Let the unknown variables be (x_1, x_2, \dots, x_n) and the quantities specified be y_1, y_2, \dots, y_n

These are related by the set of non-linear equations

$$Y_1 = f_1(x_1, x_2, \dots, x_n)$$

$$Y_2 = f_2(x_1, x_2, \dots, x_n)$$

.

.

.

.

$$Y_n = f_n(x_1, x_2, \dots, x_n)$$

To be able to solve the above equations, we start with an approximate solution $(x_1^0, x_2^0, \dots, x_n^0)$. Here, the 0 in the superscript implies the zeroth iteration in the process of solving the above equations. We need to note that the initial solution for the equations should be close to the actual solution. In other respects, the chances exist for the solution to diverge rather than converge, which reduces our chances of achieving a solution for the equations. We assume a flat voltage profile i.e $V_p = 1.0 + j0.0$ for $p = 1, 2, 3, \dots, n$; except the slack bus, which is satisfactory for almost all practical systems.

The equations are linearized about the initially assumed values. We then expand the first equation $y_1 = f_1$ and the results for the following equations.

Assuming $\Delta x_1^0, \Delta x_2^0, \dots, \Delta x_n^0$ as the corrections required for $x_1^0, x_2^0, \dots, x_n^0$ respectively for the next better solution. The equation $y_1 = f_1$ will be

$$\begin{aligned} y_1 &= f_1(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) \\ &= f_1(x_1^0, x_2^0, \dots, x_n^0) + \Delta x_1^0 \left. \frac{\partial f_1}{\partial x_1} \right|_{x^0} + \Delta x_2^0 \left. \frac{\partial f_1}{\partial x_2} \right|_{x^0} + \dots + \Delta x_n^0 \left. \frac{\partial f_1}{\partial x_n} \right|_{x^0} + \Phi_1 \end{aligned}$$

Where Φ_1 is function of higher order of Δx^8 and higher derivatives which are neglected according to N-R method. In fact this is the assumption which needs the initial solution close to the final solution. After all the equations are linearized and arranged in a matrix form, we get:

$$\begin{bmatrix} y_1 - f_1(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) \\ y_2 - f_2(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) \\ \vdots \\ y_n - f_n(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix}$$

$$\mathbf{B} = \mathbf{J} \cdot \mathbf{C}$$

Here the matrix J is called the Jacobian matrix. The solution of the equations requires calculation of the vector B on the left hand side, which is the difference of the specified quantities and calculated quantities at $(x_1^0, x_2^0, \dots, x_n^0)$. Similarly the Jacobian is calculated at this assumption. Solution of the matrix equation gives $(\Delta x_1^0, \Delta x_2^0, \dots, \Delta x_n^0)$ and the next better solution is obtained as follows:

$$\begin{aligned} x_1^1 &= x_1^0 + \Delta x_1^0 \\ x_2^1 &= x_2^0 + \Delta x_2^0 \\ &\vdots \\ x_n^1 &= x_n^0 + \Delta x_n^0 \end{aligned}$$

The better solution is now available and it is

$$(x_1^1, x_2^1, \dots, x_n^1)$$

With these values the iteration process is repeated till:

- (1) The largest element in the left column of the equations is less than the assumed value, or
- (2) The largest element in the column vector $(\Delta x_1, \Delta x_2, \dots, \Delta x_n)$ is less than assumed value.

Temporarily assuming that all buses except bus 1, are PQ buses. Thus, the unknown parameters consist of the $(n - 1)$ voltage phasors, V_2, \dots, V_n . In terms of real variables, these are:

Angles $\theta_2, \theta_3, \dots, \theta_n$ $(n - 1)$ variables

Magnitudes $|V_2|, |V_3|, \dots, |V_n|$ $(n - 1)$ variables

The linearized equations thus becomes,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \dots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \dots & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \dots & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \dots & \frac{\partial P_3}{\partial f_n} \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \dots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \dots & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \dots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \dots & \frac{\partial Q_2}{\partial f_n} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \dots & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} & \dots & \frac{\partial Q_3}{\partial f_n} \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \dots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \dots & \frac{\partial Q_n}{\partial f_n} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \Delta f_3 \\ \vdots \\ \Delta f_n \end{bmatrix}$$

In short form it can be written as,

$$\begin{bmatrix} \Delta P \\ \dots \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 \vdots J_2 \\ \dots \vdots \dots \\ J_3 \vdots J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \dots \\ \Delta f \end{bmatrix}$$

If the system consists of all kinds of buses, the above set of equations becomes,

$$\begin{bmatrix} \Delta P \\ \dots \\ \Delta Q \\ \dots \\ |\Delta V_p|^2 \end{bmatrix} = \begin{bmatrix} J_1 \vdots J_2 \\ \dots \vdots \dots \\ J_3 \vdots J_4 \\ \dots \vdots \dots \\ J_5 \vdots J_6 \end{bmatrix} \begin{bmatrix} \Delta e \\ \dots \\ \Delta f \end{bmatrix}$$

The elements of the Jacobian matrix can be derived from the three load flow equations used for N-R method.

The off-diagonal elements of J_1 are,

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}, \quad q \neq p$$

and the diagonal elements of J_1 are

$$\begin{aligned} \frac{\partial P_p}{\partial e_p} &= 2e_p G_{pp} + f_p B_{pp} - f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_p G_{pq} + f_p B_{pq}) \\ &= 2e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_p G_{pq} + f_p B_{pq}) \end{aligned}$$

The off-diagonal elements of J_2 are,

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} - f_p G_{pq}, \quad q \neq p$$

and the diagonal elements of J_2 are

$$\frac{\partial P_p}{\partial f_p} = 2f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (f_p G_{pq} + e_p B_{pq})$$

The off-diagonal elements of J_3 are,

$$\frac{\partial Q_p}{\partial e_q} = e_p B_{pq} + f_p G_{pq}, \quad q \neq p$$

and the diagonal elements are,

$$\frac{\partial Q_p}{\partial e_p} = 2e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^n (f_p G_{pq} - e_q B_{pq})$$

The off-diagonal and diagonal elements for J_4 respectively are,

$$\frac{\partial Q_p}{\partial f_q} = -e_p G_{pq} + f_p B_{pq}, \quad q \neq p$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_q G_{pq} + f_p B_{pq})$$

The off-diagonal and diagonal elements of J_5 are,

$$\frac{\partial |V_p|^2}{\partial e_q} = 0, \quad q \neq p$$

$$\frac{\partial |V_p|^2}{\partial e_p} = 2e_p$$

The off-diagonal and diagonal elements of J_6 are,

$$\frac{\partial |V_p|^2}{\partial f_q} = 0, q \neq p$$

$$\frac{\partial |V_p|^2}{\partial f_p} = 2e_p$$

The next step is that we calculate the residual column vector containing the $\Delta P, \Delta Q$ and the $|\Delta V|^2$. Let P_{sp} , Q_{sp} and $|V_{sp}|$ be the specified quantities at the bus p. Now, assuming a flat voltage profile, the value of P,Q and |V| at various buses are calculated. Then,

$$\Delta P_p = P_{sp} - P_p^0$$

$$\Delta Q_p = Q_{sp} - Q_p^0$$

$$|\Delta V_p|^2 = |V_{sp}|^2 - |V_p^0|^2$$

where the superscript zero implies that the value calculated corresponding to initial assumption i.e zeroth iteration.

After calculating the Jacobian matrix and the residual column vector corresponding to the initial solution, the desired increment vector $\begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}$ can be calculated by using any standard technique.

The next desired solution would be:

$$e_p^1 = e_p^0 + \Delta e_p^0$$

$$f_p^1 = f_p^0 + \Delta f_p^0$$

We use these voltage values in the next iteration. This process keeps repeating and the better estimates for the voltages of the buses will be:

$$\begin{aligned} e_p^{k+1} &= e_p^k + \Delta e_p^k \\ f_p^{k+1} &= f_p^k + \Delta f_p^k \end{aligned}$$

We repeat this process until the magnitude of the largest element in the residual column vector is lesser than the assumed value.

3.3.3 Newton-Raphson Algorithm

1. We assume a suitable solution for all the buses except the slack bus. We assume a flat voltage profile i.e. $V_p = 1.0 + j0.0$ for $p = 1, 2, \dots, n$, $p \neq s$, $V_s = a + j0.0$.
2. We then set a convergence criterion $= \varepsilon$ i.e. if the largest of absolute of the residues exceeds ε , the process is repeated, or else its terminated.
3. Set the iteration count $K = 0$.
4. Set the bus count $p = 1$.
5. Check if a bus is a slack bus. If that is the case, skip to step 10.
6. Calculate the real and reactive powers P_p and Q_p respectively, using the equations derived for the same earlier.
7. Evaluate $\Delta P_p^k = P_{sp} - P_p^k$
8. Check if the bus p is a generator bus. If that is the case, compare Q_p^k with the limits. If it exceeds the limits, fix the reactive power generation to the corresponding limit and treat

the bus as a load bus for that iteration and go to the next step. If lower limit is violated, set $Q_{sp}=Q_{p \min}$. If the limit is not violated evaluate the voltage residue.

$$|\Delta V_p|^2 = |V_p|_{spec}^2 - |V_p^k|^2$$

9. Evaluate $\Delta Q_p^k = Q_{sp} - Q_p^k$.
10. Increment the bus count by 1, i.e. $p = p+1$ and finally check if all the buses have been taken into consideration. Or else, go back to step 5.
11. Determine the largest value among the absolute value of residue.
12. If the largest of the absolute value of the residue is less than ϵ , go to step 17.
13. Evaluate the Jacobian matrix elements.
14. Calculate the voltage increments Δe_p^k and Δf_p^k .
15. Calculate the new bus voltage $e_p^{k+1} = e_p^k + \Delta e_p^k$ and $f_p^{k+1} = f_p^k + \Delta f_p^k$.
Evaluate $\cos \delta$ and $\sin \delta$ of all voltages.
16. Advance iteration count $K=K+1$ and go back to step 4.
17. Evaluate bus and line powers and output the results.

3.3.4 Comparison of Solution methods

The other load flow solution method we did not discuss is the Gauss method, since Gauss-Seidel method is clearly superior, because its convergence is much better. So we compare only between Newton-Raphson and Gauss-Seidel solution methods. Taking the computer memory requirement into consideration, polar coordinates are preferred for solution based on

Newton-Raphson method whereas rectangular coordinates for Gauss-Seidel method. The time taken to execute an iteration of computation is much smaller using Gauss-Seidel method in comparison to Newton-Raphson method, but if we consider the number of iterations required, Gauss-Seidel method has higher number of iterations than N-R method for a particular system, and the number of iterations increase with the increase in the size of the system. The convergence characteristics of N-R method are not affected by the selection of slack bus whereas the convergence characteristics of G-S method maybe seriously affected with the selection of the bus.

Nevertheless, the main advantage of G-S method over N-R method is the ease of programming and the efficient use of the computer memory. However, N-R method is found to be superior and more efficient than G-S method for large power systems, from the practical aspects of computational time and convergence characteristics. Even though N-R method can provide solutions to most of the practical power systems, it sometimes might fail in respect to some ill-conditioned problems.

Power Flow Analysis with STATCOM

As discussed in the earlier chapter, we use a STATCOM for transmission voltage control by shunt compensation of reactive power. Usually, STATCOM consists of a coupling transformer, a converter and a DC capacitor, as shown in the figure below.

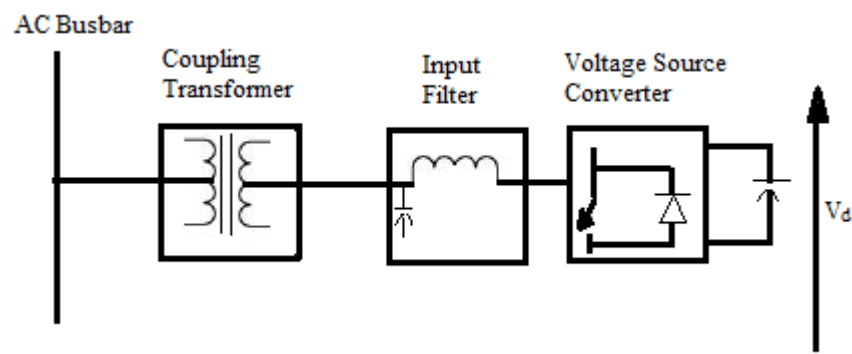


Fig 4.1

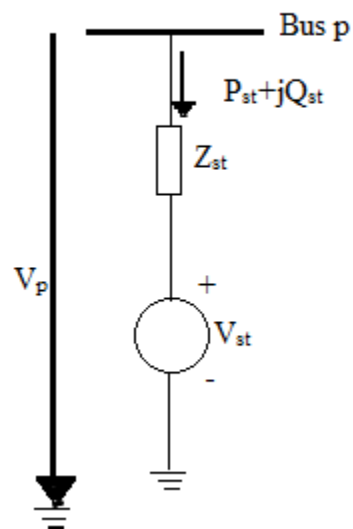


Fig 4.2

Supposing that the voltage across the statcom is $V_{st} \angle \delta_{st}$ and the voltage of the bus is $V_p \angle \delta_p$ then we have $Y_{st} = 1/Z_{st} = g_{st} + jb_{st}$

Then the power flow constraints of the statcom are given by

$$P_{st} = V_p^2 g_{st} - V_p V_{st} (g_{st} \cos(\theta_p - \theta_{st}) + b_{st} \sin(\theta_p - \theta_{st}))$$

$$Q_{st} = -V_p^2 b_{st} - V_p V_{st} (g_{st} \sin(\theta_p - \theta_{st}) - b_{st} \cos(\theta_p - \theta_{st}))$$

In our case we are using the STATCOM to control the reactive power at one of the buses to see its effect on the performance of the transmission system and infer useful conclusions from this. This is done by the control of the voltage at the required bus.

The main constraint of the STATCOM while operating is that, the active power exchange via the DC link should be zero, i.e. $P_{Ex} = \text{Re}(V_{st} I_{st}^*) = 0$.

Where

$$\text{Re}(V_{st} I_{st}^*) = V_{st}^2 g_{st} - V_p V_{st} (g_{st} \cos(\theta_p - \theta_{st}) - b_{st} \sin(\theta_p - \theta_{st}))$$

Control Function of STATCOM:

The control of the STATCOM voltage magnitude should be such that the specified bus voltage and the STATCOM voltage should be equivalent and there should be no difference between them. By proper design procedure, knowing the limits of the variables and the parameters, but not exactly knowing the power system parameters, simultaneous DC and AC control can be achieved. We can ensure the stability of the power system by the proposed STATCOM controller design. Thus it can work along with the other controllers in the network.

The bus control restraint will be

$$F = V_p - V_{sp} = 0$$

Where V_{sp} is the specified voltage for the bus.

Implementation of STATCOM to a bus network:

The Newton power flow equations for a bus system containing n number of buses, including a STATCOM are developed as follows

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \\ \Delta P_{Ex} \\ \Delta F \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \dots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \dots & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \dots & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \dots & \frac{\partial P_3}{\partial f_n} \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \dots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \dots & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \dots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \dots & \frac{\partial Q_2}{\partial f_n} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \dots & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} & \dots & \frac{\partial Q_3}{\partial f_n} \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \dots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \dots & \frac{\partial Q_n}{\partial f_n} \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \dots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \dots & \frac{\partial Q_n}{\partial f_n} \\ \frac{\partial P_{Ex}}{\partial e_2} & \frac{\partial P_{Ex}}{\partial e_3} & \dots & \frac{\partial P_{Ex}}{\partial e_n} & \frac{\partial P_{Ex}}{\partial f_2} & \frac{\partial P_{Ex}}{\partial f_3} & \dots & \frac{\partial P_{Ex}}{\partial f_n} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \Delta f_3 \\ \vdots \\ \Delta f_n \\ \Delta V_{st} \\ \Delta \delta_{st} \end{bmatrix}$$

$$B = J.C$$

Where V_{st} and δ_{st} are the two state variables of the STATCOM defined by the two equations given above. The Jacobian elements can be calculated by taking partial derivatives of the corresponding equations in the matrix. The STATCOM has two equality criterion and two state variables ΔV_{st} and $\Delta \delta_{st}$.

The first equality is the real power balancing equation, given by:

$$PE_x = \text{Re}(V_{st} I_{st}^*)$$

And the second equality for the control restraint of the STATCOM, given by:

$$F = V_p - V_{sp}$$

The Jacobian matrix elements can be found out by partially differentiating the corresponding equations.

Elements of the Jacobian Matrix:

The complex power injected at any bus p in a system is,

$$V_p I_p^* = V_p \left(\sum_{k=1}^n Y_{pk} V_k \right)^*$$

$$V_p I_p = V_p^* \left(\sum_{k=1}^n Y_{pk} V_k \right)$$

Considering

$$V_p = V_p (\cos \delta_i + j \sin \delta_i)$$

$$Y_{pk} = g_{pk} + j b_{pk}$$

$$V_k = V_k (\cos \delta_k + j \sin \delta_k)$$

Rewriting the previous equations as:

$$\begin{aligned} P_p - jQ_p &= V_p^* (Y_{p1} V_1 + Y_{p2} V_2 + \dots + Y_{pn} V_n) \\ &= V_p^* Y_{p1} V_1 + V_p^* Y_{p2} V_2 + \dots + V_p^* Y_{pn} V_n \\ &= Y_{p1} V_1 V_p \{ \cos(\delta_1 - \delta_k) + j \sin(\delta_1 - \delta_k) \} + Y_{k2} V_2 V_k \{ \cos(\delta_2 - \delta_k) \\ &\quad + j \sin(\delta_2 - \delta_k) \} + \dots + Y_{pn} V_n V_p \{ \cos(\delta_n - \delta_k) + j \sin(\delta_n - \delta_k) \} \\ &= V_1 V_k [\{ g_{k1} \cos(\delta_1 - \delta_k) - b_{k1} \sin(\delta_1 - \delta_k) \} + j \{ -g_{k1} (\delta_1 - \delta_k) + \\ &\quad b_{k1} \cos(\delta_1 - \delta_k) \}] + \dots + V_n V_k [\{ g_{kn} \cos(\delta_n - \delta_k) - b_{kn} \sin(\delta_n - \delta_k) \} \\ &\quad + j \{ -g_{kn} (\delta_n - \delta_k) + b_{kn} \cos(\delta_n - \delta_k) \}] \end{aligned}$$

Sorting out the real and imaginary parts, i.e. P_p and Q_p ,

$$P_p = V_1 V_p \{ g_{p1} \cos(\delta_1 - \delta_p) - b_{p1} \sin(\delta_1 - \delta_p) \} + \dots + V_n V_p \{ g_{pn} \cos(\delta_n - \delta_p) + b_{pn} \sin(\delta_n - \delta_p) \}$$

$$Q_p = V_1 V_p \{ g_{p1} \sin(\delta_1 - \delta_p) + b_{p1} \cos(\delta_1 - \delta_p) \} + \dots + V_n V_p \{ g_{pn} \sin(\delta_n - \delta_p) + b_{pn} \cos(\delta_n - \delta_p) \}$$

Taking the derivatives of P_p and Q_p gives us the Jacobian matrix elements:

$$\frac{\partial P_p}{\partial \delta_k} = V_p V_k (g_{pk} \sin(\delta_p - \delta_k) - b_{pk} \cos(\delta_p - \delta_k))$$

$$\frac{\partial Q_p}{\partial \delta_k} = -V_p V_k (g_{pk} \cos(\delta_p - \delta_k) + b_{pk} \sin(\delta_p - \delta_k))$$

$$\frac{\partial P_p}{\partial V_k} = V_p (g_{pk} \cos(\delta_p - \delta_k) - b_{pk} \sin(\delta_p - \delta_k))$$

$$\frac{\partial Q_p}{\partial V_k} = V_p (g_{pk} \sin(\delta_p - \delta_k) - b_{pk} \cos(\delta_p - \delta_k))$$

The elements of the Jacobian can be found out from the above equations and put in the Newton-Raphson Power Flow solution.

Stability in power system

Stability is the tendency of the power system to revert back to its original undisturbed state once the perturbations or disturbances are over. This disturbance may be caused due to a fault, or the loss of a generator, or a fault in the line, etc. The operating point of the system may change after the adjustment of the system to a new operating point post fault. This is known as the transient period and the behaviour of the system during this period is crucial in defining the stability of the system. The synchronous generators should synchronize with each other after the fault is over. This may take a lot of time if the fault is severe. During the transient period the system oscillates between multi stable points and thus it is important to damp these oscillations to bring the system to a stable operating state.

We may have two main types of stability, that is, steady state stability of the power system and transient stability.

5.1 Derivation of Swing Equations

The swing equation describes the relative motion of the rotor with respect to the synchronously rotating airgap mmf wave. The angle between the two is δ and is known as the power angle or torque angle.

Assuming synchronous operation of the generator connected to the power system, let T_e be the electromagnetic torque of the generator at synchronous speed of ω_s . During the

synchronous operation of the generator the mechanical torque is equal to the electromagnetic torque, i.e. $T_m = T_e$.

If the accelerating/decelerating torque $T_a (=T_m - T_e)$ is not equal to zero ($T_a > 0$ implying acceleration and $T_a < 0$ implying deceleration) due to a disturbance, then we have,

$$J \frac{d^2 \theta_m}{dt^2} = T_a = T_m - T_e$$

Where J is the moment of inertia for the prime mover and generator combined in Kg-m^2 . This is also known as the equation for the law of rotation.

Also, we have

$$\theta_m = \omega_s t + \delta_m,$$

Where ω_s is the constant angular velocity.

Differentiating the above equation for the angular displacement twice, we get

$$\frac{d^2 \theta_m}{dt^2} = \frac{d^2 \delta_m}{dt^2}$$

Substituting the above values in the equation for the law of rotation, we get

$$J \frac{d^2 \delta_m}{dt^2} = T_a = T_m - T_e$$

We then obtain the power equation by multiplying a factor of ω_m ,

$$J \omega_m \frac{d^2 \delta_m}{dt^2} = M \frac{d^2 \delta_m}{dt^2} = \omega_m T_m - \omega_m T_e = P_m - P_e$$

P_m is the mechanical power and P_e is the electromagnetic power.

Thus we derive the swing equation in terms of the inertia constant M as,

$$M \frac{d^2 \delta_m}{dt^2} = P_m - P_e$$

Where everything is expressed in the per unit system. And,

$$M = \frac{2H}{\omega_s}$$

Where H is the inertia constant.

5.2 Equal Area criterion

Using the swing equation, we derive the equal area criterion which can be used for stability analysis of the system. The swing equation is

$$M \frac{d^2 \delta_m}{dt^2} = P_m - P_e$$

Multiplying both sides of the equation by $2d\delta/dt$, and then integrating with respect to time, we get

$$\begin{aligned} \int 2M \frac{d^2 \delta}{dt^2} \cdot \frac{d\delta}{dt} dt &= \int 2(P_m - P_e) \frac{d\delta}{dt} dt \\ M \left(\frac{d\delta}{dt} \right)^2 &= 2 \int_{\delta_0}^{\delta} (P_m - P_e) d\delta \\ \frac{d\delta}{dt} &= \sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} (P_m - P_e) d\delta + c} \end{aligned}$$

Where δ_0 is the initial torque angle before any disturbance occurs. When $d\delta/dt = 0$ then the angle the angle δ will stop varying and the machine will be again be operating at synchronous speed post disturbance.

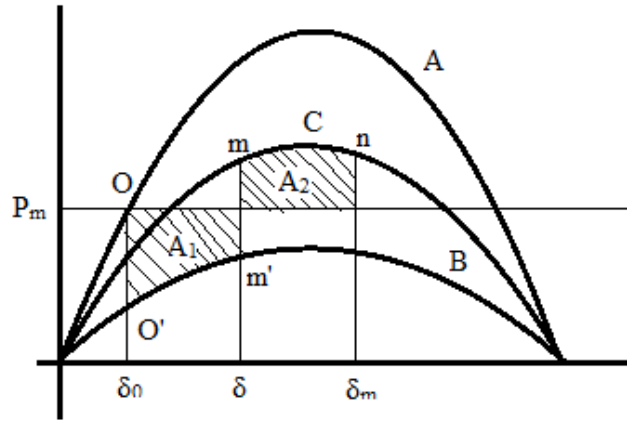


Fig 5.1

In this case, curve A represents the power angle curve corresponding to healthy condition of the system, curve B represents the fault on the line, and curve C corresponds to the situation when the faulted line is removed.

Initially for P_m the torque angle is δ_0 . At the instant of fault the output of the generator is given by O' . The rotor accelerates along curve B till the faulted line is removed at m' , when the operating point becomes m on curve C. Here the output is more than the input and thus the rotor decelerates till speed becomes equal to the speed of the bus and the torque angle ceases to increase at point n . We can conclude that the transient stability depends on the type of disturbance as well as the clearing time of the breaker.

6.1 Case Study

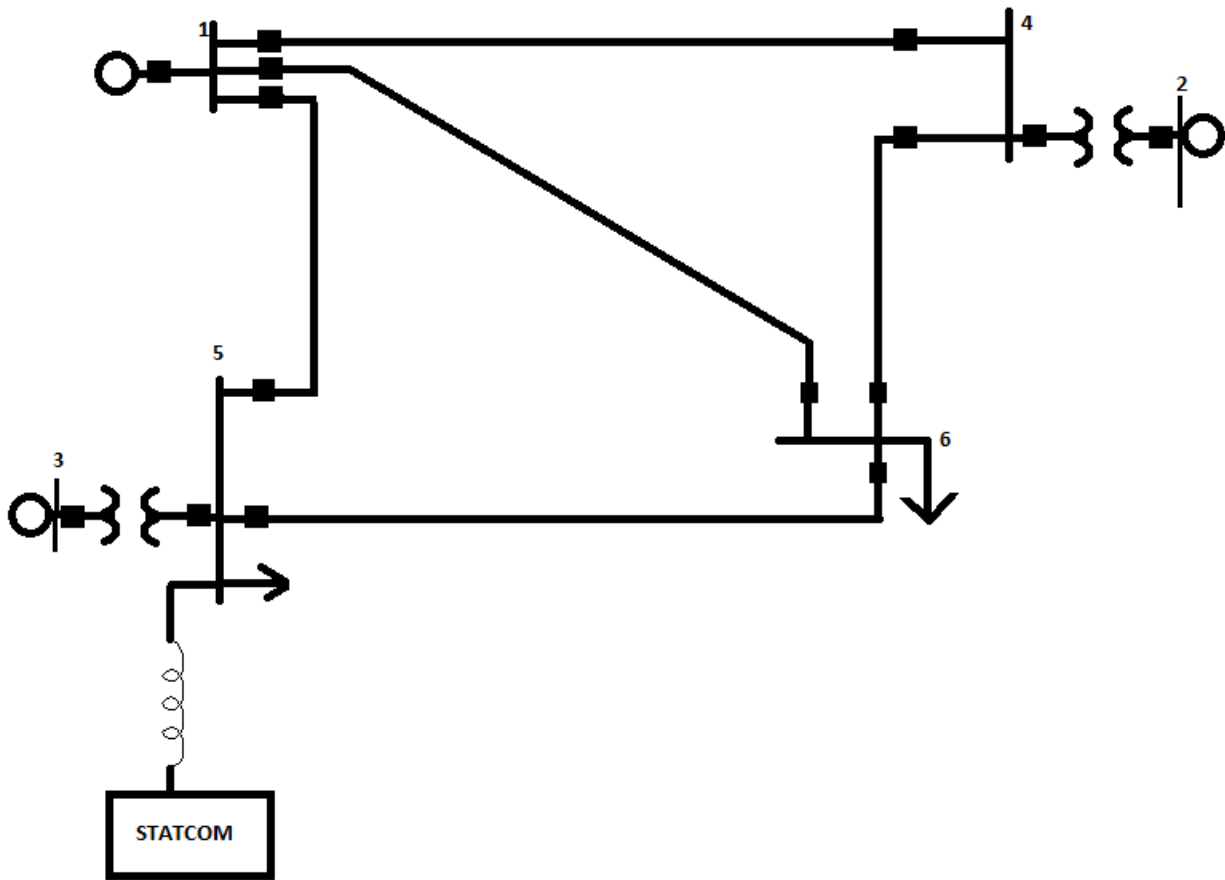


Fig 6.1

The 6 bus system shown above describes a transmission line network. The load data, voltage magnitude, generation schedule, and the reactive power limits for the buses are tabulated in the appendix. Bus 1, with a voltage specified as, is taken as the slack bus, and accounts for all the losses associated with the transmission line as well as the generators. The base MVA is taken

as 100 MVA. All the resistances, reactances, susceptances and other parameters are calculated on the basis of this MVA. First we analyse the system without the implementation of a STATCOM and see the results of the fault at different buses and removing different lines. Then we compare the graphs of the different buses with their respective faults, and find out the point where it would be best suited to implement a STATCOM. Thereafter we analyse the system with a STATCOM and check out the improvement if any. The transient stability due to the sudden fault at any point is analysed.

Case 1: Using Hadi Saadat power system analysis toolbox, we analyse the 6 bus system and for fault at bus 6 with (5,6) being the lines removed, we get

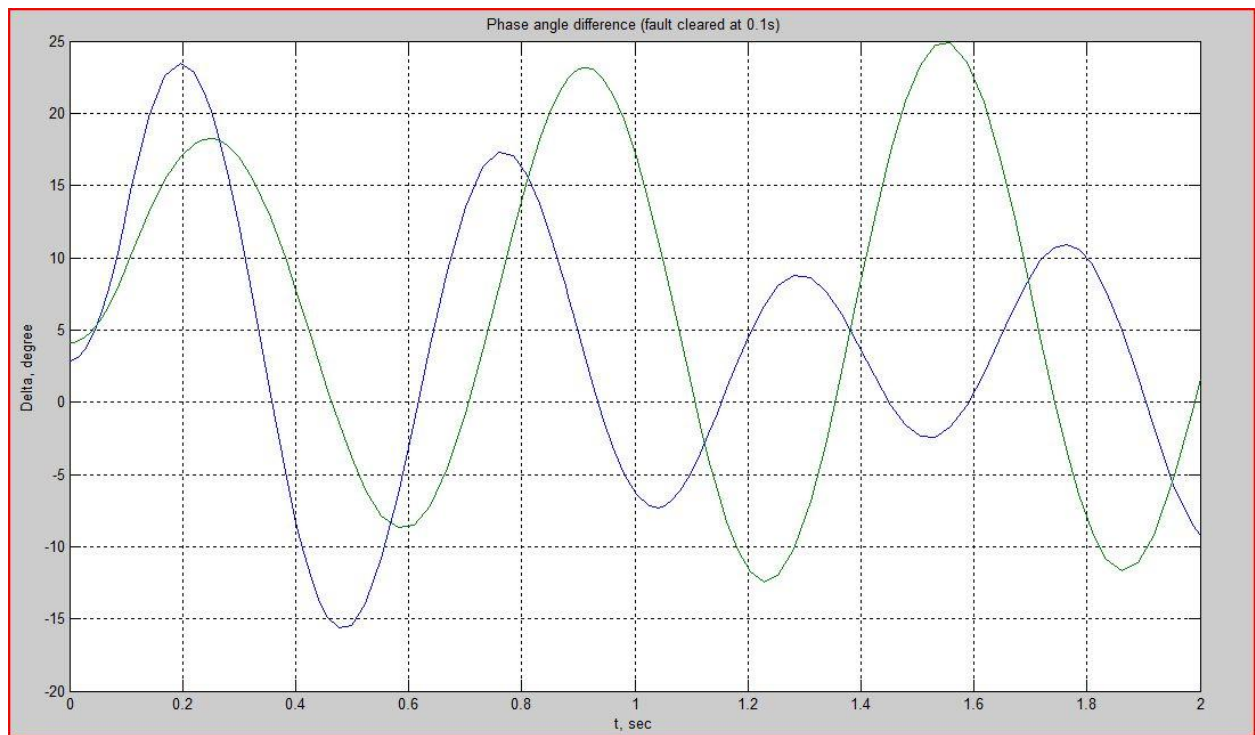


Fig 6.2

The fault is cleared at 0.1 secs for a simulation time of 2 secs.

Case 2: Similarly, we analyse for fault at bus 1 by removing lines (1,4) for the same simulation time and fault clearance time.

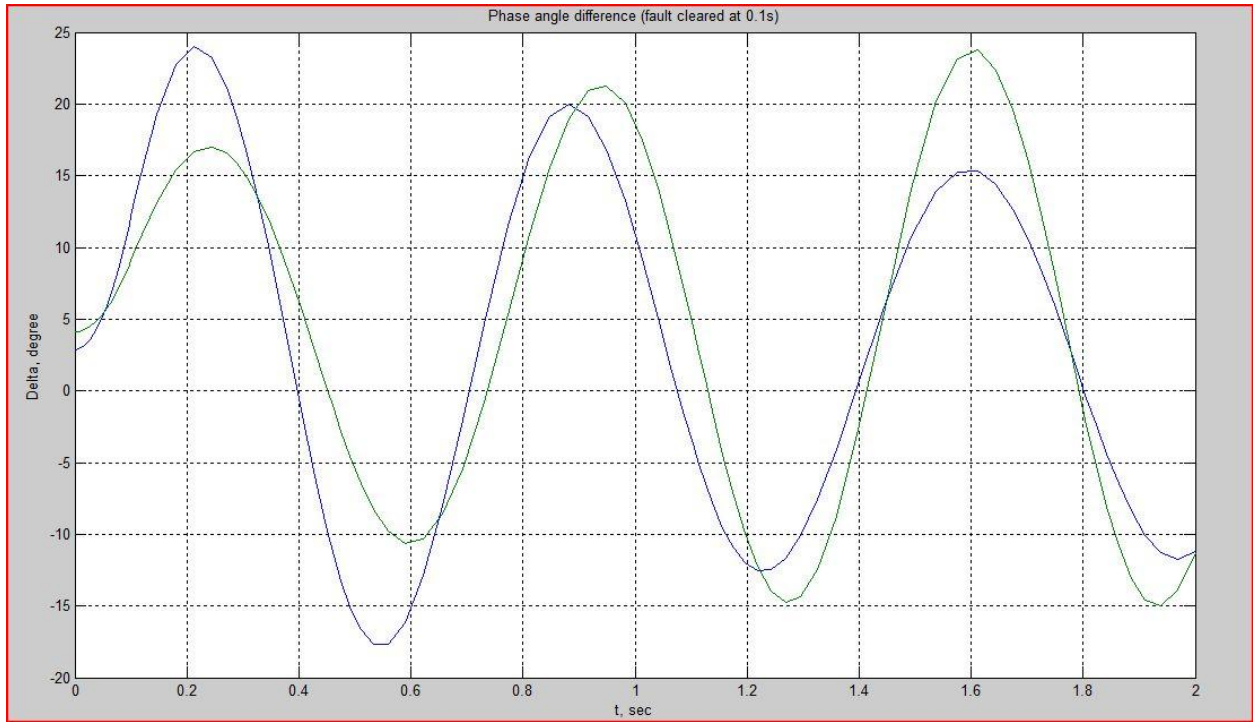


Fig 6.3

Case 3: Fault at bus 4, lines removed are (4,6)



Fig 6.4

Case 4: Fault at 6, lines removed are (1,6)

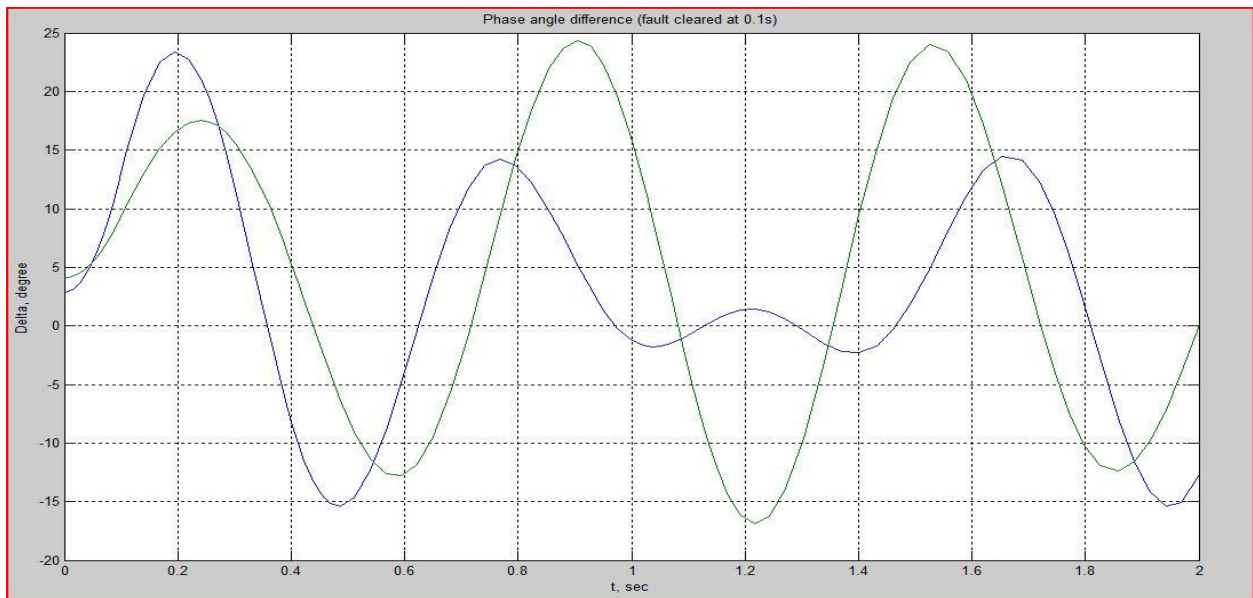


Fig 6.5

Case 5: Fault at 5, lines removed are (1,5)

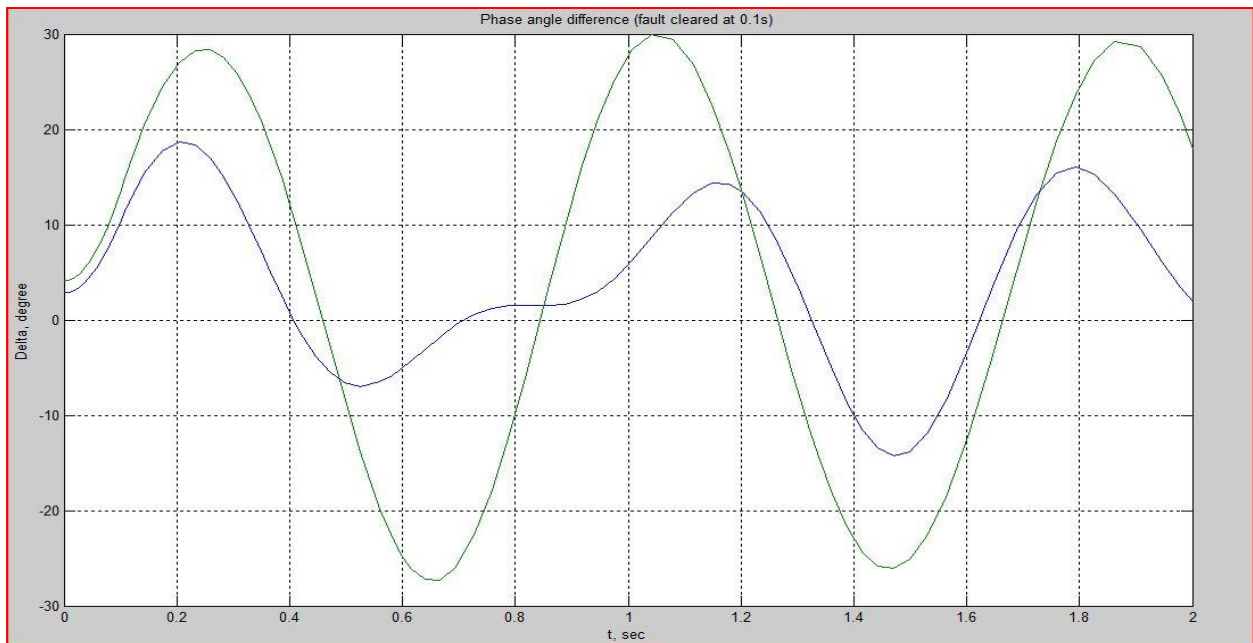


Fig 6.6

Therefore we find out that fault at bus 5 causes the maximum excursion of rotor angle of Generator 2 and 3 with respect to Generator 1. The angle of mismatch varies much in this case and it deviates much compared to the previous cases. Thus it would be an appropriate point for the implementation of STATCOM. Hence we put the STATCOM in shunt with the bus 5, and the value of STATCOM reactive power is assumed to be 30 MVar. Thus the bus 5 and STATCOM together have a total reactive power of 60 MVar.

STATCOM implementation:

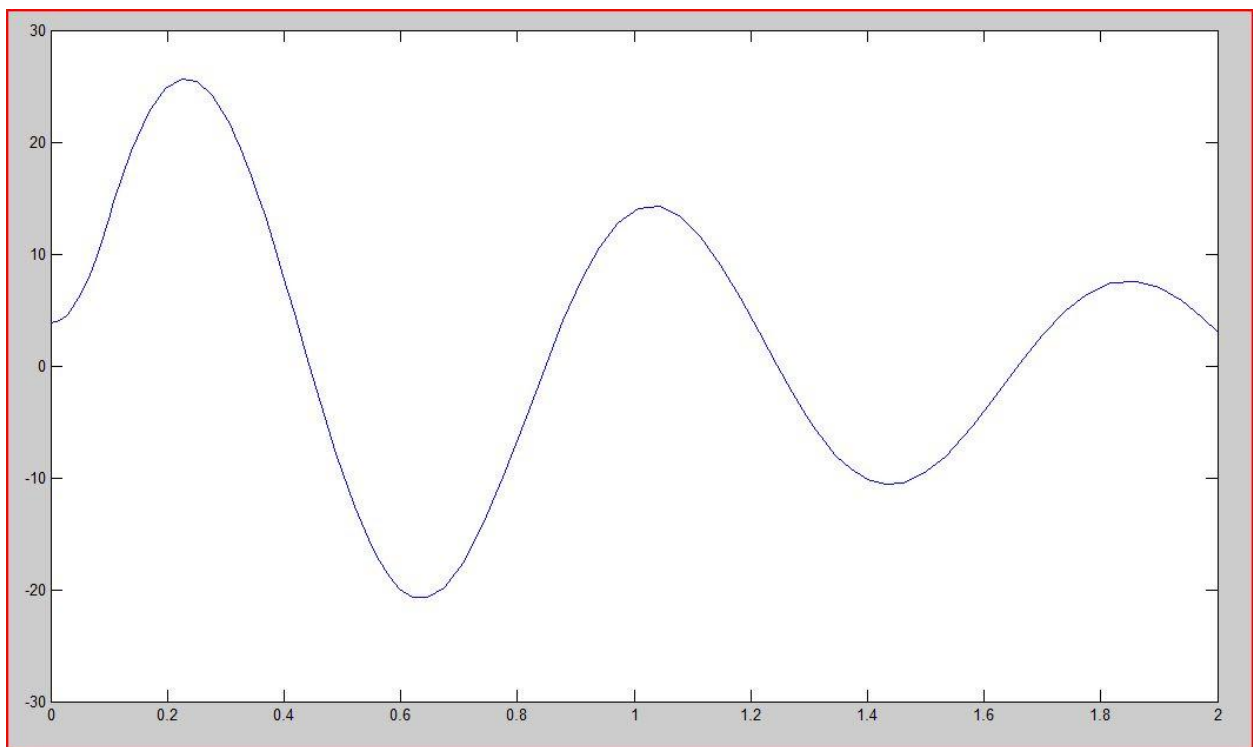


Fig 6.7

Next we find from the graphs that the bus system without and with STATCOM and see the difference in rotor angle characteristics. Improved rotor angles can be seen from both the graphs and the implementation of STATCOM has made the transient characteristics much better.

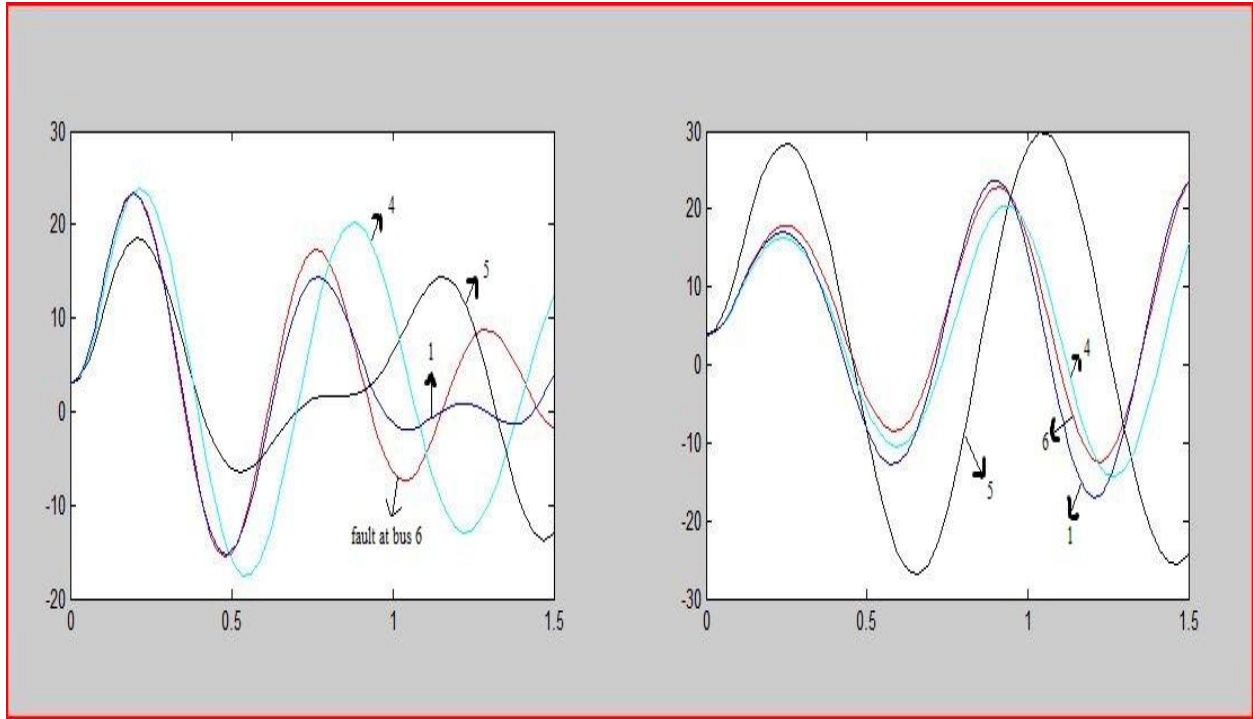


Fig 6.8

The above graph shows the difference between the behaviour of the system with faults at all the connected buses without and with the STATCOM. We can see that the waveform of the rotor angle of generator 3 with respect to generator 1 shows much better characteristics with the STATCOM and the harmonics are removed due to its action.

From the power flow solution without the STATCOM, we find out that the value of the voltage and angle at bus 5 is $1.016\angle -1.499$. Thus for reactive power control, we maintain the voltage angle of the STATCOM same as that of bus 5, while changing the magnitude of the voltage of the STATCOM to deliver or absorb reactive power.

6.2 Output

The MATLAB output for the six bus system without the STATCOM and for a fault at bus 6, with [5,6] lines removed is shown below.

Power Flow Solution by Newton-Raphson Method

Maximum Power Mismatch = 1.80187e-007

No. of Iterations = 4

Bus	Voltage	Angle	-----Load-----		---Generation---		Injected
No.	Mag.	Degree	MW	Mvar	MW	Mvar	Mvar
1	1.060	0.000	0.000	0.000	105.287	107.335	0.000
2	1.040	1.470	0.000	0.000	150.000	99.771	0.000
3	1.030	0.800	0.000	0.000	100.000	35.670	0.000
4	1.008	-1.401	100.000	70.000	0.000	0.000	0.000
5	1.016	-1.499	90.000	30.000	0.000	0.000	0.000
6	0.941	-5.607	160.000	110.000	0.000	0.000	0.000
Total			350.000	210.000	355.287	242.776	0.000

Prefault reduced bus admittance matrix

$Y_{bf} =$

$$\begin{bmatrix} 0.3517 - 2.8875i & 0.2542 + 1.1491i & 0.1925 + 0.9856i \\ 0.2542 + 1.1491i & 0.5435 - 2.8639i & 0.1847 + 0.6904i \\ 0.1925 + 0.9856i & 0.1847 + 0.6904i & 0.2617 - 2.2835i \end{bmatrix}$$

	G(i)	E'(i)	d0(i)	Pm(i)
--	------	-------	-------	-------

1	1.2781	8.9421	1.0529
---	--------	--------	--------

2	1.2035	11.8260	1.5000
---	--------	---------	--------

3	1.1427	13.0644	1.0000
---	--------	---------	--------

Enter faulted bus No. -> 6

Faulted Reduced Bus Admittance Matrix

$Y_{df} =$

$$\begin{bmatrix} 0.1913 - 3.5849i & 0.0605 + 0.3644i & 0.0523 + 0.4821i \\ 0.0605 + 0.3644i & 0.3105 - 3.7467i & 0.0173 + 0.1243i \\ 0.0523 + 0.4821i & 0.0173 + 0.1243i & 0.1427 - 2.6463i \end{bmatrix}$$

Fault is cleared by opening a line. The bus to bus nos. of the line to be removed must be entered within brackets, e.g. [5, 7]

Enter the bus to bus Nos. of line to be removed -> [5,6]

Postfault reduced bus admittance matrix

$Y_{af} =$

$$0.3392 - 2.8879i \quad 0.2622 + 1.1127i \quad 0.1637 + 1.0251i$$

$$0.2622 + 1.1127i \quad 0.6020 - 2.7813i \quad 0.1267 + 0.5401i$$

$$0.1637 + 1.0251i \quad 0.1267 + 0.5401i \quad 0.2859 - 2.0544i$$

Enter clearing time of fault in sec. $t_c = 0.1$

Enter final simulation time in sec. $t_f = 2.$

CONCLUSION

The study of the basic principles of the STATCOM is carried out as well as the basics of reactive power compensation using a STATCOM. A power flow model of the STATCOM is attempted and it is seen that the modified load flow equations help the system in better performance. The bus system shows improved plots and thus we can conclude that the addition of a STATCOM controls the output of a bus in a robust manner.

APPENDIX

The bus data and the transmission line data for the six bus system described above is given below.

basemva = 100; accuracy (convergence ϵ) = 0.0001; maximum iterations = 10;

Load Data		
Bus No	MW	Mvar
1	0	0
2	0	0
3	0	0
4	100	70
5	90	30
6	160	110

Generation Schedule				
Bus no.	Voltage Magnitude	Generation MW	MVAr Limit Min.	MVAr Limit Max.
1	1.06			
2	1.04	150	0	140
3	1.03	100	0	90

Line Data				
Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2} B$, PU
1	4	0.035	0.225	0.0065
1	5	0.025	0.105	0.0045
1	6	0.040	0.215	0.0055
2	4	0.000	0.035	0.0000
3	5	0.000	0.042	0.0000
4	6	0.028	0.125	0.0035
5	6	0.026	0.175	0.0300

Machine		Data	
Generator	R_a	X_d'	H
1	0	0.20	20
2	0	0.15	4
3	0	0.25	5

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